PROVING RELATION PROPERTIES (PROOF DESIGNS)

It is GIVEN that A is a set and R is a relation on A.

(1) Proving that relation R is reflexive:

TO PROVE: R is reflexive

Proof: Let a be any element of A. [NTS: aRa]
[or "Let $a \in A$ be given."]

⋮ [LOGICAL ARGUMENT PROVING that aRa]

∴ aRa, by definition of relation R.

∴ R is reflexive, by Direct Proof. QED

[Note: The definition of "Reflexive Relation" is a UNIVERSAL ("FOR ALL...") STATEMENT, which has been proved here using the Direct Proof method.]

Writing "∴ R is reflexive, by Direct Proof" is a shortened form of writing "∴ For all $a \in A$, aRa, by Direct Proof.
∴ Relation R is reflexive, by def'n of "Reflexive Relation."

(2) Proving that relation R is symmetric:

To Prove: R is Symmetric.

Proof: Let $a \in A$ and $b \in A$ be given.
[or, "Let a and b be any elements of A."]

Suppose aRb. [NTS: bRa]

⋮ ← [LOGIC PROVING THAT bRa.]

∴ bRa, by def'n of relation R.

∴ R is Symmetric, by Direct Proof. QED.

See the Note on the next page regarding the last statement.

Proving Relation Properties (Continued)

[Note: The definition of "Symmetric Relation" is a UNIVERSAL CONDITIONAL STATEMENT, which has been proved here using the Direct Proof Method.]

WRITING "∴ R is symmetric, by Direct Proof" is a shortened form of writing "∴ For all $a \in A$ and $b \in B$, if aRb, then bRa, by Direct Proof.
∴ R is symmetric by def'n of 'Symmetric Relation'."

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③ Proving that relation R is transitive

<u>To Prove</u>: R is transitive :

<u>Proof</u>: Let a, b and c be any elements of A.
Suppose aRb and bRc. [NTS: aRc].

⋮ [LOGIC PROVING THAT aRc.]

∴ aRc, by definition of R.
∴ R is transitive, by Direct Proof. QED.

[NOTE: The definition of "transitive relation" is a universal conditional statement, which has been proved using the Direct Proof method.]

WRITING "∴ R is transitive, by Direct Proof" is a shortened form of WRITING "∴ For all $a, b, c \in A$, if aRb and bRc, then aRc, by Direct Proof.
∴ R is transitive, by definition of "Transitive Relation."

<u>Note</u>: After the statement "Suppose aRb" has been written, you will conclude that elements a and b satisfy the conditions that are listed in the definition of "Relation R". When you do this, you write
[Since aRb] ∴ "a and b satisfy .... , <u>by definition of relation R</u>."